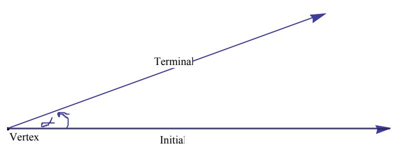
Business PreCalculus MATH 1643 Section 004, Spring 2014

Lesson 28: Angels and Their Measure

Definition 1. **Ray:** *A **ray** is a part of a line made up of a point, called the **endpoint**, and all of the points on one side of the endpoint.*

Definition 2. **Angle:** *An **angle** is formed by ROTATING a ray about its endpoint. The angle's **initial side** is the ray's original position. The angle's **terminal side** is the ray's final position after rotation. The endpoint is called the **vertex** of the angle.*



Definition 3. **Positive and Negative Angles:** *The curved arrow drawn near the vertex indicates both the **direction and amount** of rotation. **IF** the rotation is **counterclockwise**, the result is a **positive angle**. **IF** the rotation is **clockwise**, the result is a **negative angle**.*

Definition 4. **Coterminal Angles:** *Angles that have the same initial and terminal sides are called **coterminal** angles.*

Definition 5. **Standard Position:** *An angle in a rectangular coordinate is in **standard position** if its vertex is at the origin and its initial side is the positive x-axis.*

Definition 6. **Measuring Angels by Using Degrees:** *A measure of **one degree**, denoted by* 1° *is assigned to an angle resulting from a rotation* $\frac{1}{360}$ *of a complete revolution counterclockwise. An **acute angle** has measure between* 0° *and* 90°*. An **obtuse angle** has measure between* 90° *and* 180°*. A **right angle** has measure* 90°*. A **straight angle** has measure* 180°*.*

Definition 7. **Relationship between Degrees, Minutes and Seconds:**

$$1' = (\frac{1}{60})^\circ \qquad 1^\circ = 60' \qquad 1^\circ = (3600)''$$

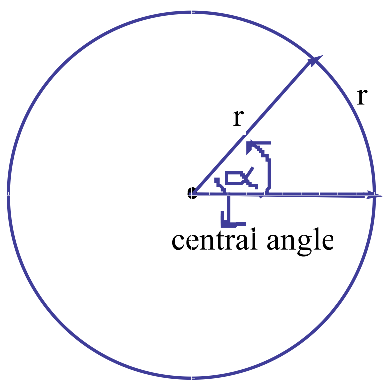
$$1'' = (\frac{1}{3600})^\circ \qquad 1'' = (\frac{1}{60})' \qquad 1' = 60''$$

Example 1. *Convert $24°8'15''$ to decimal degree notation.*

Solution:

$$\begin{aligned} 24°8'15'' &= 24° + 8 \times 1' + 15 \times 1'' \\ &= 24° + 8(\frac{1}{60})° + 15(\frac{1}{3600})° \\ &\approx 24.14° \end{aligned}$$

Definition 8. **Radian Measure:** *An angle whose center is at the center of a circle is called a **central angle**. Since the length of arc is equal to the radius then $\alpha = 1$ **radian**.*



Definition 9. **Radian Measure of a Central Angle:** *The radian measure α of a central angle that intercepts an arc of length s on a circle of radius r is given by*

$$\alpha = \frac{r}{s} radians.$$

Definition 10. **Converting between Degrees and Radians:** *We have the relationship* **$180°$ = π radians**.

$$1° = \frac{\pi}{180}\, radians \qquad \theta° = \theta(\frac{\pi}{180})\, radians$$

$$1 radian = \frac{180}{\pi}\, degrees \qquad \theta\, radians = \theta(\frac{180}{\pi})\, degrees$$

Example 2. *Convert $-225°$ to radians.*

Solution:

$$-225° = -225°(\frac{\pi}{180}) = \frac{-225\pi}{180} = -\frac{5\pi}{4}\, radians$$

Definition 11. **Complements and Supplements Angles:** *Two positive angles are **complements** if their sum is $90°$. Two positive angles are **supplements** if their sum is $180°$.*

Example 3. *Find the complement and the supplement of $73°$.*

Solution: *If θ represents the complement of $73°$, then $73+\theta = 90$; so $\theta = 17°$ is the complement of $73°$.*

If α represents the supplement for $73°$, then $73 + \alpha = 180$; so $\alpha = 107°$ is the supplement of $73°$.

Definition 12. **Arc Length Formula:** *The formula for the length s of the arc intercepted by a central angle with* ***radian*** *measure θ in a circle of radius r is*

$$s = r\theta.$$

Example 4. *A circle has a radius of 18 inches. Find the length of the arc intercepted by a central angle with measure 210°.*

Solution: *We must first convert the central angle measure from degrees to radians. So $\theta = 210^\circ = 210(\frac{\pi}{180}) = \frac{7\pi}{6}$. Then*

$$s = r\theta = 18(\frac{7\pi}{6}) = 21\pi \approx 65.97\, inches$$

Definition 13. **Linear and Angular Speed:** *Suppose an object travels around a circle of radius r. If the object travels through a central angle of θ radians and an arc of length s, in time t, then*

1. *$\nu = \frac{s}{t}$ is the* ***linear speed*** *of the object.*
2. *$\omega = \frac{\theta}{t}$ is the* ***angular speed*** *of the object.*

Moreover, we can relate the linear speed to the angular speed and write $\nu = r\omega$.

Example 5. *A model plane is attached to a swivel so that it flies in a circular path at the end of a 12-foot wire at the rate of 15 revolutions per minute. Find the angular speed and the linear speed of the plane.*

Solution: *Because angular speed is measure in radians per unit of time, we first convert revolutions per minute into radians per minute. Recall that, $1\, revolution = 2\pi\, radians$. Then $15\, revolutions = 15(2\pi) = 30\pi\, radians$. So the* ***angular speed*** *is $\omega = 30\pi$ radians per minute. The linear speed is given by*

$$\nu = r\omega = 12(30\pi) \approx 1131\, feet\ per\ minute.$$

Definition 14. **Area of a Sector:** *The* ***area*** *A of a sector of a circle of radius r formed by a central angle with* ***radian*** *measure θ is given by*

$$A = \frac{1}{2}r^2\theta.$$

Example 6. *How many square inches of pizza have you eaten if you eat a sector of an 18-inch-diameter pizza whose edges form a 30° angle?*

Solution: *First, we have to* ***convert*** *30° to radians to use the sector area formula. So $\theta = 30(\frac{\pi}{180}) = \frac{\pi}{6}$. The pizza's radius is $r = \frac{diameter}{2} = \frac{18}{2} = 9$. Hence*

$$\begin{aligned} A &= \frac{1}{2}r^2\theta \\ &= \frac{1}{2}(9)^2(\frac{\pi}{6}) \\ &\approx 21\, square\ inches. \end{aligned}$$